Time Variable Cosmological Constants from the Age of Universe

Lixin Xu*,† Jianbo Lu, and Wenbo Li
Institute of Theoretical Physics, School of Physics & Optoelectronic Technology,
Dalian University of Technology, Dalian, 116024, P. R. China

In this paper, time variable cosmological constant, dubbed *age cosmological constant*, is investigated motivated by the fact: any cosmological length scale and time scale can introduce a cosmological constant or vacuum energy density into Einstein's theory. The age cosmological constant takes the form $\rho_\Lambda = 3c^2 M_P^2/t_\Lambda^2$, where t_Λ is the age of our universe or conformal time. The effective equation of state of age cosmological constant are $w_\Lambda^{eff} = -1+\frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c}$ and $w_\Lambda^{eff} = -1+\frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c}(1+z)$ when the age of universe and conformal time are taken as the role of cosmological time scales respectively. They are the same as the so-called agegraphic dark energy models. However, the evolution history are different from the agegraphic ones for their different evolution equations.

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I. INTRODUCTION

The observation of the Supernovae of type Ia [1, 2] provides the evidence that the universe is undergoing accelerated expansion. Jointing the observations from Cosmic Background Radiation [3, 4] and SDSS [5, 6], one concludes that the universe at present is dominated by 70% exotic component, dubbed dark energy, which has negative pressure and push the universe to accelerated expansion. Of course, a natural explanation to the accelerated expansion is due to a positive tiny cosmological constant. Though, it suffers the so-called *fine tuning* and *cosmic coincidence* problems. However, in 2σ confidence level, it fits the observations very well [7]. If the cosmological constant is not a real constant but is time variable, the fine tuning and cosmic coincidence problems can be removed. In fact, this possibility was considered in the past years.

In particular, the dynamic vacuum energy density based on holographic principle are investigated extensively [8, 9]. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite and has relations with the area of its boundary. By applying the principle to cosmology, one can obtain the upper bound of the entropy contained in the universe. For a system with size L and UV cut-off Λ without decaying into a black hole, it is required that the total energy in a region of size L should not exceed the mass of a black hole of the same size, thus $L^3\rho_\Lambda \le LM_P^2$. The largest L allowed is the one saturating this inequality, thus $\rho_\Lambda = 3c^2M_P^2L^{-2}$, where c is a numerical constant and M_P is the reduced Planck Mass $M_P^{-2} = 8\pi G$. It just means a *duality* between UV cut-off and IR cut-off. The UV cut-off is related to the vacuum energy, and IR cut-off is related to the large scale of the universe, for example Hubble horizon, future event horizon or particle horizon as discussed by [8, 9, 10, 11]. The holographic dark energy in Brans-Dicke theory is also studied in Ref. [13, 14, 15, 16, 17, 18].

Another dark energy model with relations with holographic dark energy, named agegraphic dark energy, was also researched extensively recently [19, 20, 21]. This model is based on the application of the well-known Heisenberg uncertainty relation to the universe. Therefore, the energy density of metric fluctuations of Minkowski space-time is $\rho_\Lambda \sim M_P^2/t^2$, where t is time or length scale. Obviously, it looks like the holographic one. Indeed, there some relations between them [19].

As known, for any nonzero value of the cosmological constant Λ, a natural length scale and time scale

$$r_\Lambda = t_\Lambda = \sqrt{3/|\Lambda|} \tag{1}$$

can be introduced into Einstein's theory. Reversely, any cosmological length scale and time scale can introduce a cosmological constant or vacuum energy density into Einstein's theory. For a positive cosmological constant, one has

$$\Lambda(t) = \frac{3}{r_\Lambda^2(t)} = \frac{3}{t_\Lambda^2}. \tag{2}$$

* Corresponding author
†Electronic address: lxxu@dlut.edu.cn

When a dynamic time scale is taken, a time variable cosmological constant can be obtained. Obviously, a natural time scale is the age of our universe. Inspired by this observation, we can consider time variable cosmological constant from this analogue and let the holographic principle and Heisenberg uncertainty relation alone. For its explicit relation with the age of the universe, we dub it *age cosmological constant*.

This paper is structured as follows. In Section II, we give a brief review of a time variable cosmological constant. In Section III, time variable cosmological constants–age cosmological constants–are investigated, where the age of our universe and conformal time are taken as the role of time scales. Section IV are conclusions.

II. TIME VARIABLE COSMOLOGICAL CONSTANT

The Einstein equation with a cosmological constant is written as

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+\Lambda g_{\mu\nu}=8\pi GT_{\mu\nu}, \tag{3}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of ordinary matter and radiation. From the Bianchi identity, one has the conservation of the energy-momentum tensor $\nabla^{\mu}T_{\mu\nu}=0$, it follows necessarily that Λ is a constant. To have a time variable cosmological constant $\Lambda=\Lambda(t)$, one can move the cosmological constant to the right hand side of Eq. (3) and take $\tilde{T}_{\mu\nu}=T_{\mu\nu}-\frac{\Lambda(t)}{8\pi G}g_{\mu\nu}$ as the total energy-momentum tenor. Once again to preserve the Bianchi identity or local energy-momentum conservation law, $\nabla^{\mu}\tilde{T}_{\mu\nu}=0$, one has, in a spacial flat FRW universe,

$$\dot{\rho}_{\Lambda}+\dot{\rho}_m+3H\left(1+w_m\right)\rho_m=0, \tag{4}$$

where $\rho_{\Lambda}=M_P^2\Lambda(t)$ is the energy density of time variable cosmological constant and its equation of state is $w_{\Lambda}=-1$, and w_m is the equation of state of ordinary matter, for dark matter $w_m=0$. It is natural to consider interactions between variable cosmological constant and dark matter [11], as seen from Eq. (4). After introducing an interaction term Q, one has

$$\dot{\rho}_m+3H\left(1+w_m\right)\rho_m=Q, \tag{5}$$

$$\dot{\rho}_{\Lambda}+3H\left(\rho_{\Lambda}+p_{\Lambda}\right)=-Q, \tag{6}$$

and the total energy-momentum conservation equation

$$\dot{\rho}_{tot}+3H\left(\rho_{tot}+p_{tot}\right)=0. \tag{7}$$

For a time variable cosmological constant, the equality $\rho_{\Lambda}+p_{\Lambda}=0$ still holds. Immediately, one has the interaction term $Q=-\dot{\rho}_{\Lambda}$ which is different from the interactions between dark matter and dark energy considered in the literatures [22] where a general interacting form $Q=3b^2H\left(\rho_m+\rho_{\Lambda}\right)$ is put by hand. With observation to Eq. (6), the interaction term Q can be moved to the left hand side of the equation, and one has the effective pressure of variable cosmological constant, dark energy

$$\dot{\rho}_{\Lambda}+3H\left(\rho_{\Lambda}+p_{\Lambda}^{eff}\right)=0 \tag{8}$$

where $p_{\Lambda}^{eff}=p_{\Lambda}+\frac{Q}{3H}$ is the effective dark energy pressure. Also, one can define the effective equation of state of dark energy

$$\begin{aligned} w_{\Lambda}^{eff} &= \frac{p_{\Lambda}^{eff}}{\rho_{\Lambda}} \\ &= -1+\frac{Q}{3H\rho_{\Lambda}} \\ &= =-1-\frac{1}{3}\frac{d\ln\rho_{\Lambda}}{d\ln a}. \end{aligned} \tag{9}$$

The Friedmann equation as usual can be written as, in a spacial flat FRW universe,

$$H^2=\frac{1}{3M_P^2}\left(\rho_m+\rho_{\Lambda}\right). \tag{10}$$

III. AGE COSMOLOGICAL CONSTANTS

A. Age of the universe as time scale

The age of the universe is defined as

$$t_\Lambda = \int_0^t dt' = \int_0^a \frac{da'}{a'H}. \tag{11}$$

Taking it as the role of time scale, one has the vacuum energy density

$$\rho_\Lambda = 3c^2 M_P^2 / t_\Lambda^2, \tag{12}$$

where c is the model constant. Defining the dimensionless energy densities $\Omega_m = \rho_m/(3M_P^2 H^2)$ and $\Omega_\Lambda = \rho_\Lambda/(3M_P^2 H^2)$, the Friedmann equation is rewritten as

$$\Omega_m + \Omega_\Lambda = 1. \tag{13}$$

The energy conservation equation (4) can be rewritten as

$$\frac{d\ln H}{dx} + \frac{3}{2}\left(1 - \Omega_\Lambda\right) = 0, \tag{14}$$

where $x = \ln a$. Adjoining Eq. (11), Eq. (12) and the definition of the dimensionless energy density Ω_Λ, one has

$$\int_0^a \frac{d\ln a'}{H} = \frac{c}{H}\sqrt{\frac{1}{\Omega_\Lambda}}. \tag{15}$$

Taking the derivative with respect to $x = \ln a$ from the both sides of the above equation (15), one has the differential equation

$$\frac{d\ln H}{dx} + \frac{1}{2}\frac{d\ln \Omega_\Lambda}{dx} + \frac{\sqrt{\Omega_\Lambda}}{c} = 0. \tag{16}$$

Substituting Eq. (14) into above differential equation, one obtains the differential equation of Ω_Λ

$$\Omega_\Lambda' = \Omega_\Lambda\left(3 - 3\Omega_\Lambda - \frac{2}{c}\sqrt{\Omega_\Lambda}\right), \tag{17}$$

where $'$ denotes the derivative with respect to $x = \ln a$. This equation describes the evolution of the dimensionless energy density of dark energy. Clearly, one can see that this equation is different from the corresponding one derived in [19], given the initial conditions, and the evolution of the age cosmological constant is different from the one of agegraphic dark energy. With the derivative of the variable of redshift z, the above equation can be rewritten as

$$\frac{d\Omega_\Lambda}{dz} = -\Omega_\Lambda\left[3\left(1 - \Omega_\Lambda\right) - \frac{2\sqrt{\Omega_\Lambda}}{c}\right](1+z)^{-1}. \tag{18}$$

From Eq. (9), it is easy to obtain the effective equation of state of dark energy

$$\begin{aligned} w_\Lambda^{eff} &= -1 - \frac{1}{3}\frac{d\ln\rho_\Lambda}{d\ln a} \\ &= -1 + \frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c}. \end{aligned} \tag{19}$$

It is in the range of $-1 < w_\Lambda^{eff} < -1 + 2/(3c)$, when one notices the dark energy density rasio $0 \le \Omega_\Lambda \le 1$. The form of the effective equation of state of the horizon cosmological constant is different from the one of agegraphic dark energy. In the earlier time where $\Omega_\Lambda \approx 0$, one has $w_\Lambda^{eff} \to -1$. It means at earlier time, the conformal time cosmological constant behaves just like a true cosmological constant. When the age cosmological constant dominates, its property depends on the parameter c strongly. One can also easily have the deceleration parameter

$$\begin{aligned} q &= -\frac{\dot{H} + H^2}{H^2} \\ &= -1 - \frac{d\ln H}{d\ln a} \\ &= \frac{1}{2} - \frac{3}{2}\Omega_\Lambda. \end{aligned} \tag{20}$$

To have an current accelerated expansion of the universe, $\Omega_{\Lambda 0} > 1/3$ is required. In Fig. 1, the evolution curves with respect to redshift z is plotted with different values of parameter c when the initial value $\Omega_{\Lambda 0} = 0.70$ is taken.

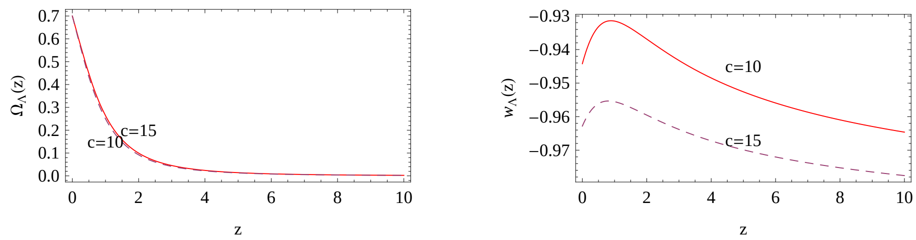

FIG. 1: The evolutions of dimensionless density parameter $\Omega_\Lambda(z)$ (right panel) and effective equation of state $w_\Lambda(z)$ (left panel) of age cosmological constant with respect to the redshift z, where the values $\Omega_{\Lambda 0} = 0.70$, $c = 10$ (solid lines), $c = 15$ (dashed lines) are adopted.

B. Conformal time as time scale

The conformal time is defined as

$$\eta_\Lambda = \int_0^t \frac{dt'}{a} = \int_0^a \frac{da'}{a'^2 H}. \tag{21}$$

In this case, the vacuum energy density is given as

$$\rho_\Lambda = 3c^2 M_P^2/\eta_\Lambda^2. \tag{22}$$

Repeating the analysis and calculations as done in III A, one has the differential equation of Ω_Λ

$$\Omega'_\Lambda = \Omega_\Lambda \left[3 - 3\Omega_\Lambda - \frac{2\sqrt{\Omega_\Lambda}}{ca} \right], \tag{23}$$

where $'$ denotes the derivative with respect to $x = \ln a$. With the derivative of the variable of redshift z, the above equation can be rewritten as

$$\frac{d\Omega_\Lambda}{dz} = -\Omega_\Lambda \left[3\left(1-\Omega_\Lambda\right)\left(1+z\right)^{-1} - \frac{2\sqrt{\Omega_\Lambda}}{c} \right]. \tag{24}$$

The effective equation of state and deceleration parameter are given as

$$w_\Lambda^{eff} = -1 + \frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c}(1+z), \tag{25}$$

$$q = \frac{1}{2} - \frac{3}{2}\Omega_\Lambda. \tag{26}$$

These equations are different from the ones derived in [20]. It is easy to see that the behavior of conformal time cosmological constant is rather different from that of age cosmological constant. At later time where $a \to \infty$, the effective equation of state goes to $w_\Lambda^{eff} \to -1$, it mimics a true cosmological constant regardless of the value of c. The evolution curves with respect to redshift z is plotted in Fig. 2, where the different values of parameter c and the initial value $\Omega_{\Lambda 0} = 0.70$ are taken respectively.

IV. CONCLUSIONS

In this paper, time variable cosmological constants, dubbed *age cosmological constants*, are investigated inspired by the observations that any nonzero value of the cosmological constant Λ can introduce a natural length scale and time scale into Einstein's theory. Reversely, a variable cosmological time or length scale can introduce a time variable

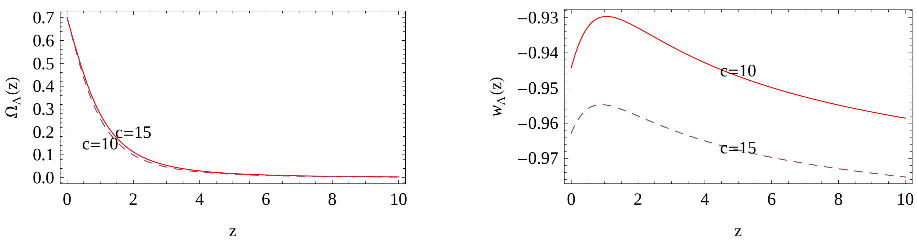

FIG. 2: The evolutions of dimensionless density parameter $\Omega_\Lambda(z)$ (right panel) and effective equation of state $w_\Lambda(z)$ (left panel) of age cosmological constant with respect to the redshift z, where the values $\Omega_{\Lambda 0} = 0.70$, $c = 10$ (solid lines), $c = 15$ (dashed lines) are adopted.

positive tiny cosmological constant. Here the age of our universe and conformal time were used as time scales. The results are rather different from that of the so-called agegraphic dark energy models, though the effective equation of state of age cosmological constants are common. But for the different evolution equations of the dimensionless energy density, please see Eq.(17) and Eq.(23) (or Eq.(18) and Eq.(24)). So, the whole evolution history will be different from that of agegraphic dark energy models. In Fig. 1 and Fig. 2, the evolution curves were plotted with different values of the model parameter c and the same initial condition $\Omega_{\Lambda 0} = 0.7$. It can be seen that the model only contains the model parameter c, here we just put some values of the parameter c and leave the 'precise' value to be obtained by fitting cosmic observations such as type Ia supernovae, CMB and BAO etc.

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